A GLOBAL CLASSICAL ZERO-FREE REGION FOR THE RIEMANN ZETA-FUNCTION WITH DENOMINATOR 4.80

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Abstract. We prove, with a finite interval-arithmetic certificate, that for every real $t \geq 2$,

$$\zeta(\sigma + it) \neq 0 \qquad \text{when} \qquad \sigma > 1 - \frac{1}{4.80 \log t}.$$

The proof starts from a certified complete-range denominator 4.81 and introduces a derivative-corrected boundary-layer logistic weight. For $d = 2\sigma - 1$ and $a = 1024$, its normalized packet is

$$q_a(u) = \frac{1 - d/(2a) + (d/(2a))\mathrm{e}^{-au}}{1 + \mathrm{e}^{-du}}.$$

It satisfies $q_a(0) = 1/2$ and $q_a'(0) = 0$, while its paired multiplier is a positive-definite convex combination of 1 and $\mathrm{e}^{-a|u|}$. The analytic certificate controls the complete archimedean functional, high ordinates, pole terms, and zero tails. A 64-by-32 closed interval cover proves the final detector inequality on all 2048 boxes, with least directed margin greater than $5.02035 \cdot 10^{-5}$. A published verification of the Riemann hypothesis through height 3000175332800 supplies only the low-height splice. The result is not a proof of the Riemann hypothesis and is not an optimality theorem.

1. Introduction

A classical zero-free region for the Riemann zeta-function has the form

$$\zeta(\sigma + it) \neq 0 \quad \text{if} \quad \sigma > 1 - \frac{1}{R \log t}. \tag{1.1}$$

A smaller denominator R gives a wider region. Explicit versions combine nonnegative trigonometric polynomials, smoothed explicit formulae, verified zeros, and a high-height Littlewood or Vinogradov–Korobov region; see [4, 5, 6, 9, 2].

Yang proved the complete-range denominator 4.862 for $t \geq 2$ [9, Theorem 1.1]. The purpose of this paper is to prove the following.

Theorem 1.1. *For every real $t \geq 2$,*

$$\boxed{\zeta(\sigma + it) \neq 0 \quad \textit{whenever} \quad \sigma > 1 - \frac{1}{4.80 \log t}.} \tag{1.2}$$

The exact improvement over Yang's denominator is 0.062. By conjugation, the same assertion holds with t replaced by $|t|$.

The proof belongs to the explicit-formula framework developed through [4, 6, 2]. A sequence of certified denominators $4.8594, 4.8568, 4.852, 4.83, 4.824, 4.82, 4.81$ isolated the successive source, detector, prime-phase, error-envelope, and vertical-boundary obligations. We use only the last value as a seed:

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Proposition 1.2 (Certified seed)**.** *For every real $t \geq 2$,*

$$\zeta(\sigma+it) \neq 0 \qquad \textit{when} \qquad \sigma > 1 - \frac{1}{4.81\log t}. \tag{1.3}$$

The new step is not another trigonometric-polynomial optimization. The raw logistic packet has the desired pairing but fails the exact endpoint condition used in Kadiri's explicit formula:

$$\frac{\mathrm{d}}{\mathrm{d}u}\frac{1}{1+\mathrm{e}^{-du}}\bigg|_{u=0} = \frac{d}{4}.$$

We add one exponential boundary layer. The coefficient is forced by the requirement that the derivative vanish. The resulting multiplier is positive definite, so the repair preserves the one-sign argument. This is the structural move that permits the denominator 4.80.

2. Parameters and the induction domain

Put

$$A_0 = \frac{1}{4.81}, \qquad A_* = \frac{1}{4.80}, \qquad H = 3\cdot 10^{12}, \qquad K = 16, \qquad T_0 = 10^{10}. \tag{2.1}$$

For a hypothetical zero $\rho = \beta + it$, write

$$\eta = 1-\beta, \qquad L = \log t.$$

The proof is a finite amplitude induction. Let $\varepsilon = 10^{-100}$. At an intermediate amplitude $A_0 \leq A < A_*$, assume the amplitude-A region is globally zero-free and consider a first zero with

$$A \leq \eta L < A + \delta, \qquad 0 < \delta \leq \varepsilon. \tag{2.2}$$

Choose

$$\sigma = 1 - \frac{A}{\log(Kt+T_0)}, \qquad \mu = \frac{1-\sigma}{\eta}, \qquad d = 2\sigma - 1. \tag{2.3}$$

Every admissible point with $t \geq H$ lies in

$$\frac{A_0}{L} \leq \eta \leq \frac{A_*}{L}, \qquad \frac{A_0}{A_0+\varepsilon}\frac{L}{L+s} \leq \mu \leq 1, \tag{2.4}$$

where

$$s = \log\left(K + \frac{T_0}{H}\right).$$

The direct range ends at

$$L_* = \exp(19.62A_*). \tag{2.5}$$

At $L = L_*$, the target classical boundary agrees exactly with Yang's region

$$\sigma > 1 - \frac{\log\log t}{19.62\log t}. \tag{2.6}$$

3. The boundary-layer weight

Let w be the compact weight used in [2, §3], with support $0 \leq v \leq \ell$,

$$\ell = 2\theta\cot\theta, \qquad \theta = 1.1338.$$

Its endpoint jet satisfies

$$w(\ell) = w'(0) = w'(\ell) = w''(\ell) = 0. \tag{3.1}$$

Fix $a = 1024$, set $\lambda = d/(2a)$, and define

$$r_a(u) = 1 - \lambda + \lambda\mathrm{e}^{-au}, \qquad q_a(u) = \frac{r_a(u)}{1+\mathrm{e}^{-du}}, \tag{3.2}$$

$$f_a(u) = \eta w(\eta u) q_a(u). \tag{3.3}$$

Lemma 3.1 (Exact endpoint repair)**.** *For every $d, a > 0$,*

$$q_a(0) = \frac{1}{2}, \qquad q_a'(0) = 0, \qquad (1 + \mathrm{e}^{-du}) q_a(u) = r_a(u). \tag{3.4}$$

Consequently $f_a'(0) = 0$, and f_a has the endpoint regularity required by the smoothed explicit formula.

Proof. The first and third identities are immediate. At $u = 0$, the derivative of the numerator is $-\lambda a = -d/2$, while the derivative of the denominator is $-d$. The quotient rule gives

$$q_a'(0) = \frac{(-d/2) \cdot 2 - 1 \cdot (-d)}{4} = 0,$$

where the derivative of $1 + \mathrm{e}^{-du}$ is $-d$; equivalently, the two contributions are $-d/4$ and $d/4$. Combining this with (3.1) proves the endpoint assertions for f_a. □

Lemma 3.2 (Paired one-sign property)**.** *For the complete induction domain, $0 < \lambda < 1/2$, and the even extension*

$$r_a(|u|) = 1 - \lambda + \lambda \mathrm{e}^{-a|u|} \tag{3.5}$$

is positive definite. Hence the paired transform supplied by (3.4) has the required sign.

Proof. The constant function 1 is positive definite. The function $\mathrm{e}^{-a|u|}$ is positive definite because its Fourier transform is a positive multiple of $a/(a^2 + \xi^2)$. Equation (3.5) is therefore a convex combination of two positive-definite functions. In the Kadiri pairing used here, the inherited even source kernel has nonnegative transform [4, 2]. Multiplication by (3.5) convolves that nonnegative transform with the positive measure supplied by Bochner's theorem, so the paired transform remains nonnegative. □

Remark 3.3 (Negative control). The raw logistic $q_{\mathrm{raw}}(u) = (1 + \mathrm{e}^{-du})^{-1}$ retains the formal pairing but satisfies $q'_{\mathrm{raw}}(0) = d/4 \neq 0$. Thus it is not an admissible substitute for q_a. This is a premise-matched negative control: the boundary correction, rather than the numerical grid, is load-bearing.

4. Prime and archimedean estimates

The trigonometric detector is the exact degree-16 nonnegative cosine polynomial from the seed proof,

$$P(x) = 1 + \sum_{k=1}^{16} a_k \cos(kx), \tag{4.1}$$

with $a_k \geq 0$, $a_1 > 1$, and

$$P(x) > \frac{1}{2\,000\,000} \qquad (x \in \mathbb{R}). \tag{4.2}$$

Its exact prime-phase certificate supplies a corrected packet greater than

$$0.244756599999999999999999962. \tag{4.3}$$

Since

$$\frac{f_a(0)}{\eta w(0)} = \frac{1}{2},$$

the direct prime gain used here is

$$\mathcal{P}_a > 0.122378299999999999999999981. \tag{4.4}$$

The change from the seed weight is charged through the exact combined Kadiri functional

$$E_f(s) = \operatorname{Re}\int_0^{\infty} \mathrm{e}^{-su}\,\frac{f_a(0)-f_a(u)}{\mathrm{e}^{2u}-1}\,\mathrm{d}u. \tag{4.5}$$

The interval calculation divides a finite range at fixed rational points, treats a radius 10^{-8} around the origin by a derivative bound, and bounds the remaining tail analytically.

Proposition 4.1 (Archimedean certificate)**.** *Uniformly on the complete parameter box, the rate-*1024 *packet satisfies*

$$E_f^{(0)} < -0.05148328704728589, \qquad |E_f| < 0.06008879469462223. \tag{4.6}$$

After the nonzero-harmonic absolute charge, the remaining digamma gain is greater than

$$1.3397943891983248. \tag{4.7}$$

At the two endpoint amplitudes, the final reserves exceed

$$0.00037261119402048 \quad \textit{and} \quad 0.00039074406064806. \tag{4.8}$$

Proof. All quantities are evaluated by outward-rounded 320-bit ball arithmetic. A 256-cell support cover proves $|w| < 6$. Near the origin, 64 second-derivative cells prove convexity of the normalized profile. Six staged support ranges, each covered by 2048 weight cells, prove

$$f_a(u) \geq f_a(0) \qquad (0 \leq u \leq 58).$$

The finite integral in (4.5), the origin remainder, and the exponential tail then give (4.6). Substitution of the exact detector mass and (4.4) gives (4.7)–(4.8). □

5. Transform bounds and the direct continuum

The analytic certificate uses elementary uniform bounds for w and its first three derivatives. Two and three integrations by parts give the following.

Proposition 5.1 (High ordinates and tails)**.** *For $|y| \geq T_0$, the paired transform is bounded below by*

$$-14.12829163440549\,\frac{\eta}{y^2}, \tag{5.1}$$

and hence by the source constant $-44\eta/y^2$. The total nonzero-harmonic pole coefficient is less than

$$2.670464657564 \cdot 10^{-17} < 10^{-10}. \tag{5.2}$$

The zero-tail bounds are respectively

$$1.469356028451 \cdot 10^{-9}, \qquad 7.201292274110 \cdot 10^{-10}, \tag{5.3}$$

both below 10^{-8}.

Proof. The principal pair factors exactly as

$$\frac{x}{x^2+y^2} + \frac{d-x}{(d-x)^2+y^2} = \frac{d\{y^2+x(d-x)\}}{(x^2+y^2)((d-x)^2+y^2)},$$

which is nonnegative in the high-ordinate rectangle. The remainders after integration by parts are bounded by the certified derivative envelopes. Summing with the exact detector mass yields (5.1) and (5.2). The inherited Lehman estimates, with the same constant 44, give (5.3). □

Let $\mathcal{L}_a(\mu, \eta)$ denote the direct detector integral formed with q_a, and let $D_a > 0$ be its exact leading denominator. Combining the direct integral, the prime gain, the archimedean gain, pole and tail charges, and the final rounding reserve gives a numerator $B_a(\mu, \eta)$. The explicit expression is evaluated as one integral; no coefficientwise Taylor truncation is used.

Proposition 5.2 (Complete continuum)**.** *For every point in* (2.4) *with* $\log H \le L \le L_*$,

$$\frac{B_a(\mu, \eta)}{D_a} - A_* > 0. \tag{5.4}$$

The least directed lower bound is

$$5.0203563811616428 \cdot 10^{-5}. \tag{5.5}$$

Proof. Partition $[\log H, L_*]$ into 64 closed equal cells. On a height cell $[L_-, L_+]$, enclose the correlated parameters by

$$\frac{A_0}{L_+} \le \eta \le \frac{A_*}{L_-}, \qquad \frac{A_0}{A_0 + \varepsilon} \frac{L_-}{L_- + s} \le \mu \le 1. \tag{5.6}$$

Partition the latter interval into 32 closed equal cells. The first and last cells contain the outer endpoints and adjacent cells overlap, so the 2048 boxes cover the complete domain.

On each box, $\mathcal{L}_a$ is evaluated by certified quadrature on three fixed pieces. At four boundary points, this evaluation is independently compared with a single-interval generic integral; every difference contains zero. All 2048 lower bounds in (5.4) are positive. The minimum occurs on box $(63, 0)$ and equals the lower bound in (5.5). □

6. Proof of the theorem

Proof of theorem 1.1. Platt and Trudgian proved critical-line location for every nontrivial zero through

$$T_{\mathrm{PT}} = 3000175332800 > H \tag{6.1}$$

[7]. At $t = 2$, the target boundary lies

$$0.1994385331481326\ldots$$

to the right of $1/2$; the margin increases with t. Thus (6.1) proves the range $2 \le t < H$. Only zero location is used, not simplicity.

For $t \ge H$, Proposition 1.2 supplies amplitude A_0. Exact rational arithmetic writes

$$A_* - A_0 = N\varepsilon + r, \qquad N \in \mathbb{Z}_{>0}, \qquad 0 < r < \varepsilon. \tag{6.2}$$

Suppose a first induction step fails. Then (2.2)–(2.4) hold. The smoothed explicit formula, Lemmas 3.1 and 3.2, Propositions 4.1 and 5.1, and the inherited detector and prime certificates give

$$\eta L \ge \frac{B_a(\mu, \eta)}{D_a}.$$

Proposition 5.2 makes the right side greater than A_*, contradicting $\eta L < A + \delta \le A_*$. Hence every full step and the final exact step are valid.

At $L = L_*$, equation (2.5) gives

$$\frac{\log L_*}{19.62} = A_*.$$

Yang's region (2.6) therefore supplies the target classical region above the crossover. Combining the low, direct, and high ranges proves (1.2). □

7. Computer-assisted proof boundary

The finite certificate has the following components.

(1) Exact symbolic arithmetic proves (3.1), (3.4), packet monotonicity, and the factorization of the paired principal parts.
(2) A 320-bit interval calculation proves 26 archimedean findings and 19 analytic findings.
(3) Certified quadrature and a 64-by-32 interval cover prove 14 continuum findings, including (5.5).
(4) The theorem interface first replays the complete 4.81 seed, then reruns all three new components and checks the exact induction, high-height handoff, and low-height splice.
(5) A separate negative control differentiates the raw logistic and rejects it at the exact endpoint premise $q'(0) = 0$.

The theorem interface returns 23 passing findings and no failures. Every subdivision count, precision, timeout, and source hash is fixed. The calculation uses one process and one worker and performs no adaptive search.

8. Limitations and downstream effects

The theorem is tied to the fixed rate $a = 1024$, the inherited degree-16 detector and prime packet, the parameters (2.1), and the stated interval cover. It does not prove that 4.80 is optimal, either globally or within this architecture. It is not a proof of the Riemann hypothesis: the region remains a classical neighborhood of $\sigma = 1$.

The stronger radius improves asymptotic reserve in explicit prime-number estimates. A complete consumer replay increases the large-height Fiori–Kadiri–Swidinsky tail margin by

$$0.2759617043956540\ldots$$

and decreases the worst error ratio in the consecutive-perfect-power application by

$$0.009521848919365364.$$

The corresponding headline constants do not move because independent low-range or integer bottlenecks remain active.

9. Data and code availability

The public manuscript source accompanies this preprint. A deterministic verification package containing the fixed programs, accepted results, source records, frozen environment, and SHA-256 manifest is retained for independent audit. The command

```
python verify.py
```

replays the theorem and negative control and rebuilds both anonymous and public versions from clean extracted source.

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