The global classical zeta zero-free denominator improves to 4.81

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Abstract

The classical global zero-free region for the Riemann zeta-function is widened to denominator 4.81 for every $t \geq 2$. The proof combines a sharp exact vertical boundary, an everywhere-positive rational degree-16 detector, exact prime-phase certificates, a directed interval induction, and the published Platt–Trudgian finite-height theorem.

$$\zeta(\sigma + it) \neq 0 \quad \left(t \geq 2,\ \sigma > 1 - \frac{1}{4.81 \log t}\right)$$

1 The global region

For every real $t \geq 2$, the zeta-function has no zero to the right of $1 - 1/(4.81 \log t)$. A smaller denominator gives a wider zero-free region.

The same-range certified denominator improves from 4.824 to 4.81. Yang's published denominator is 4.862.

$$\zeta(\sigma + it) \neq 0 \qquad \left(\sigma > 1 - \frac{1}{4.81 \log t}\right)$$

2 A sharp accessible boundary

The former vertical one-sign boundary 151/153 is inaccessible at the target starting height 10^{10}. The proof replaces it with an exact rational boundary whose degree-six positivity polynomial has no nonnegative root.

At the target height the sharp boundary has positive access margin, while the former boundary has negative margin. A premise-matched counterfeit restores only the former boundary and is rejected.

$$c_* = \frac{46110325513857}{46729244180480}$$

$$d_0 - c_* > 4.57 \times 10^{-5}, \qquad d_0 - \frac{151}{153} < 0$$

3 Exact detector and source gains

An inward rationalization of the optimized degree-16 cosine detector is proved positive for every real phase. For each prime below 100, the verifier encloses every root of the corresponding exact degree-240 Chebyshev gap polynomial.

The resulting prime packet, cutoff correction, and sign-directed digamma estimate provide the positive source gains used by the induction.

$$P(x) > \frac{1}{2\,000\,000} \quad \text{for every real } x$$

$$\sum_{p<100} (\log p) \min_x G_p(1, x) > 0.2447566$$

4 Finite induction and global splice

The shifted explicit-formula losses are enclosed by seven certified quadrature packets and outward-rounded to $26.421\eta^2 + 247.723\eta^3$.

All 2048 closed induction boxes have positive directed margin. Exact finite steps reach denominator 4.81, Yang's Littlewood region supplies the high-height handoff, and the published Platt–Trudgian theorem covers the low-height range.

$$\frac{B(\mu, \eta)}{D} - \frac{1}{4.81} > 3.9690 \times 10^{-5}$$

$$3000175332800 - 3000000000000 = 175332800$$

5 What the certificate checks

The public package hash-pins the theorem, source, counterfeit, propagation, polynomial, and quadrature layers. It separates the exact published critical-line endpoint from the rounded simplicity statement and uses no zero-simplicity premise.

The theorem, counterfeit, propagation, and propagation-counterfeit modes all pass at their declared bounds. Optimized Python mode refuses because the certificate requires assertions.

6 Pinned certificate

The pinned package proves the exact detector floor, all prime-phase root enclosures, the sharp boundary, seven shifted error packets, every induction box, both global splices, and the complete noncircular downstream propagation.

Run command

```
nice -n 19 gtimeout 1200s env PYTHONDONTWRITEBYTECODE=1 OMP_NUM_THREADS=1 OPENBLAS_NUM_THREADS=
```

Pinned files

- `canon/witnesses/C-0122/verify.py`
- `canon/witnesses/C-0122/rational_candidate.py`
- `canon/witnesses/C-0122/prime_phase.py`
- `canon/witnesses/C-0122/boundary.py`
- `canon/witnesses/C-0122/error_envelope.py`
- `canon/witnesses/C-0122/counterfeit.py`
- `canon/witnesses/C-0122/propagation_verify.py`
- `canon/witnesses/C-0122/propagation_counterfeit.py`
- `canon/witnesses/C-0122/PIN.md`
- `canon/claims/C-0122-global-classical-zeta-zero-free-denominator-4-81.md`

7 Scope

The certificate proves denominator 4.81 for every real $t \geq 2$. It does not claim the denominator is globally optimal.

8 Sources

- Canonical claim: `canon/claims/C-0122-global-classical-zeta-zero-free-denominator-4-81.md`
- Proof: `canon/witnesses/C-0122/PROOF.md`
- Source and verifier pin: `canon/witnesses/C-0122/PIN.md`
- Published finite-height theorem: D. J. Platt and T. S. Trudgian, The Riemann hypothesis is true up to $3 \cdot 10^{12}$, Bulletin of the London Mathematical Society 53 (2021), 792–797. `https://doi.org/10.1112/blms.12460`